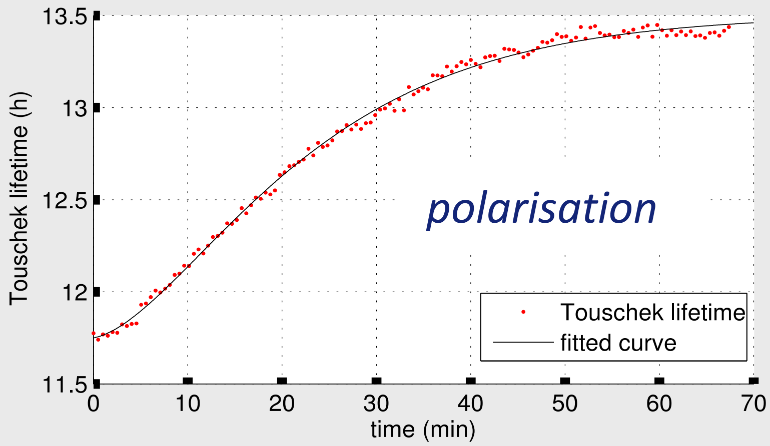
Modeling of spin depolarisation

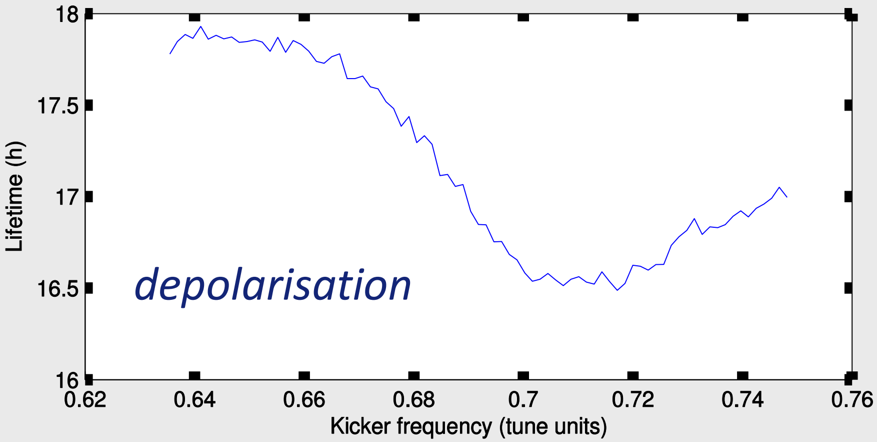
Nicola Carmignani, Friederike Ewald, Boaz Nash

The difficulty to find a sharp spin depolarsiation resonance at the ESRF (see DEELS 2014) motivated a careful analysis of the depolarisation process. A spin tracking code was developed by collegues in the Beam Dynamics Group. It allows to follow the electron spins of many particles as they propagate in the storage ring lattice while being excited by an oscillating magnetic field. The output of the code is the polarisation of the electron beam after N turns in the storage ring. Simulations were done for the ESRF and the Australian Synchrotron. The results reveal substantial differences in the depolarisation behaviour of the two storage rings in accordance with the experimental findings. We would be interested in simulating the depolarisation at other light sources and compare the results with measurements in order to validate the code and get a deeper understanding on which parameters are favourable for the detection of distinct spin depolarisation resonances.

Questions we had after many measurements

(see also DEELS 2014 presentation)





Theoretical: $\tau_p = 15.6\,\text{min}$ $\quad \frac{\Delta\tau_t}{\tau_t} = 0.151$

Measured: $\tau_p = 15.9 \pm 0.6\,\text{min}$ $\quad \frac{\Delta\tau_t}{\tau_t} = 0.150 \pm 0.005$

The beam may be depolarized within a broad range of many kHz, whatever we do.
Why don't we see narrow resonances at the synchrotron tune and its side bands ?

Simple simulations suggest extremely narrow resonance widths.
This is in opposition to our experimental findings.
What is wrong about our understanding / simulation of the resonance width ?

Development of a spin tracking code ("FESTA")

B. Nash + N. Carmignani: Code FESTA (**F**ast **E**lectron **S**pin **T**racking based on **A**T)

AT lattice

↓ ↓

Fast Ring

Orbit Angle Matrix (OAM) ("spin response function")

↓ ↓

Orbit tracking for Np = 100 particles and N_t ~10^6 turns

phase space coordinates →

Spin tracking:

1. spin tune (dipoles)
2. kicker induced spin angle variation
3. effect of quadrupoles (from OAM)

→ spin vector of each electron

← Compute average polarization for each frequency

Which effects influence the spin motion and resonance BW?

0. **All electrons have the same energy and precess about the vertical axis at $\nu_{sp,0}$ = a γ_0 = 13.707**

   -- > *Very narrow depolarisation resonance expected*

   -- > *Depolarisation will occur, when kicker imparts a total angle of $\pi/2$, which leads to a depolarisation in N_{d0} turns:*

$$N_{d0} = \frac{\pi^2}{4\theta_k \nu_{sp}}$$

   = 180 000 turns (= 0.5 s @ ESRF) for θ_k = 1 μrad kick strength

1. **Energy spread σ_δ is added**

   -- > *This causes a spread in spin tune $\sigma_{\nu,sp} = \nu_{sp} \sigma_\delta$*

   -- > *Depolarsiation in a broad range covering the spin tune spread, directly linked to spread in electron energy σ_δ*

Which effects influence the spin motion and resonance BW?

2. Effect of synchrotron oscillations <---> analogy to motional narrowing in NMR

Longitudinal component of the spin modulated by synchrotron oscillations

$$\Rightarrow \quad S_z(t) = A\cos\left(\omega_0 \nu_{sp} t + \frac{\omega_0 a \gamma_0 \delta_0}{\omega_s}\sin(\omega_s t)\right) \qquad (1)$$

$$= A\sum_{n=-\infty}^{\infty} J_n(B)\cos(\omega_c + n\omega_m)t$$

with

$$B = \frac{\omega_0 a \gamma_0 \delta_0}{\omega_s} = \frac{\nu_{sp,0}\delta_0}{\nu_s}$$

B is also known as spin tune modulation index [*] and can be interpreted as the number of sidebands inside the spin tune spread.

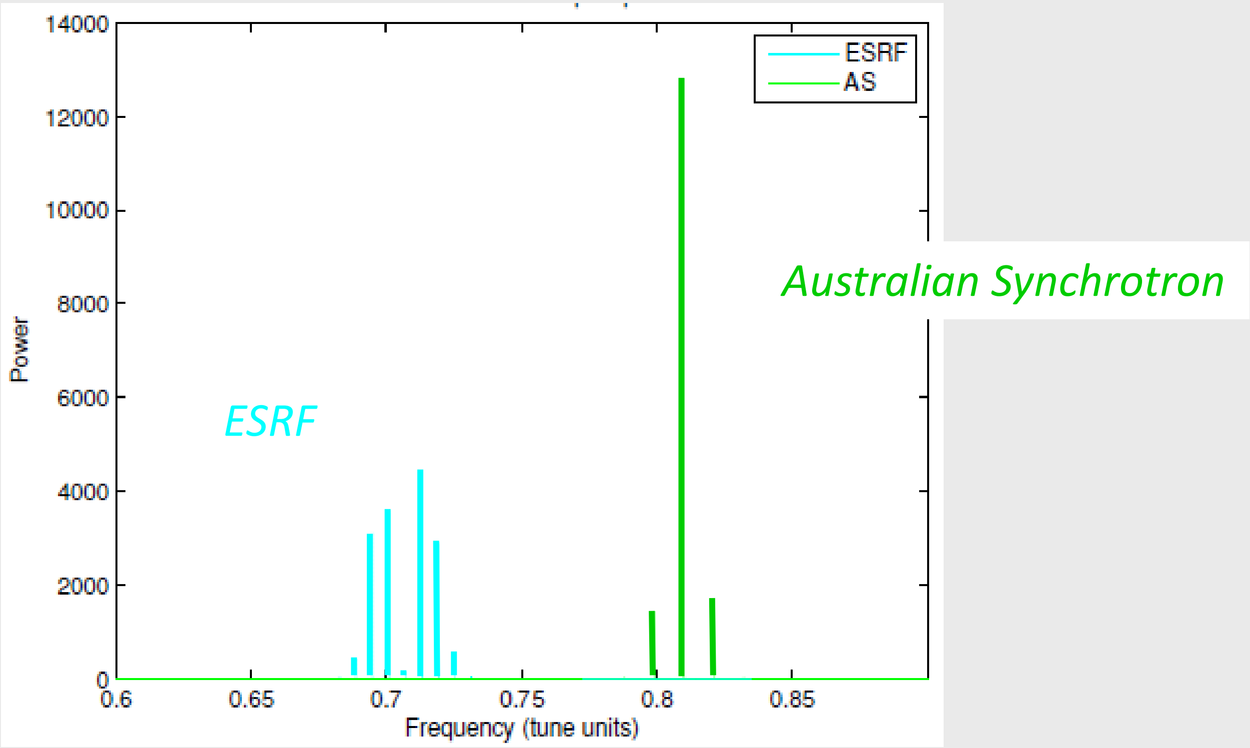
-- > FFT of (1) --> Narrow resonance width with synchrotron sidebands -- >

[*] J. Buon, "A stochastic model of Depolarisation Enhancement due to Large Energy Spread in Electron Storage Rings", LAL-RT 88-13 (1988)

Which effects influence the spin motion and resonance BW?

2. *Effect of synchrotron oscillations <---> analogy to motional narrowing in NMR*

-- > FFT --> Narrow resonance width with synchrotron sidebands



-- > sideband spacing is determined by ratio between spin tune and synchrotron tune

-- > sideband amplitudes are modulated as $J_n(B)$

Which effects influence the spin motion and resonance BW?

2. Effect of synchrotron oscillations -- Detailed:

Spin tune is modulated by synchrotron oscillations ω_s: $\nu_{sp} = a\gamma_0(1+\delta_0\cos(\omega_s t))$

Spin vector oscillates with $S_z(t) = A\cos\varphi(t)$

The phase φ must fulfill $d\varphi/dt = \omega_{sp} = \omega_0\nu_{sp}$

$$\Rightarrow \quad \varphi(t) = \omega_0\nu_{sp}t + \frac{\omega_0 a\gamma_0\delta_0}{\omega_s}\sin(\omega_s t)$$

with the relation for a carrier signal (frequency ω_c) modulated by a frequency ω_m

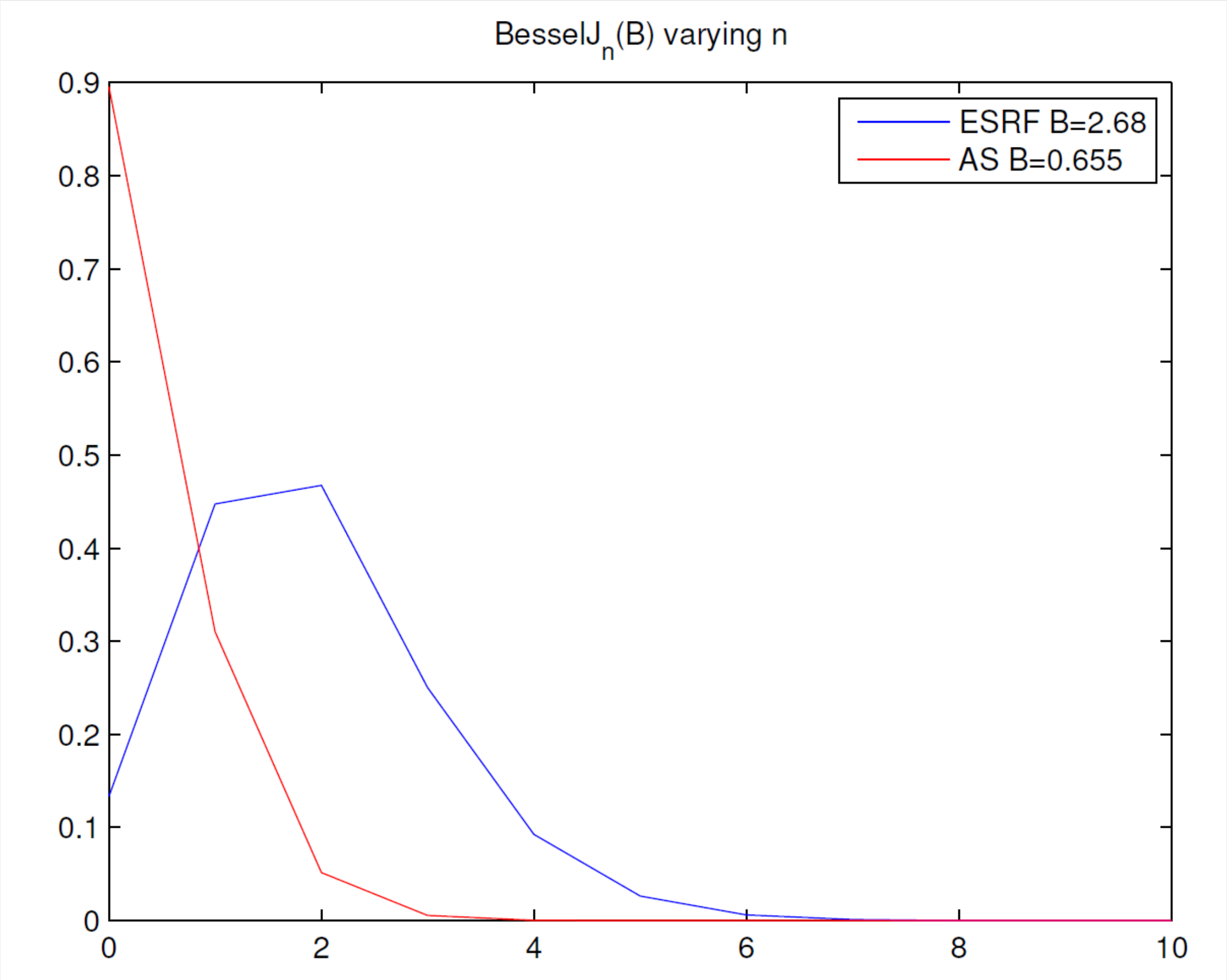
$$\cos(\omega_c t + B\sin\omega_m t) = \sum_{n=-\infty}^{\infty} J_n(B)\cos(\omega_c + n\omega_m)t$$

we can identify

$$B = \frac{\omega_0 a\gamma_0\delta_0}{\omega_s} = \frac{\nu_{sp,0}\delta_0}{\nu_s}$$

Bessel function

Bessel function $J_n(B)$ for B-values of ESRF and ALS



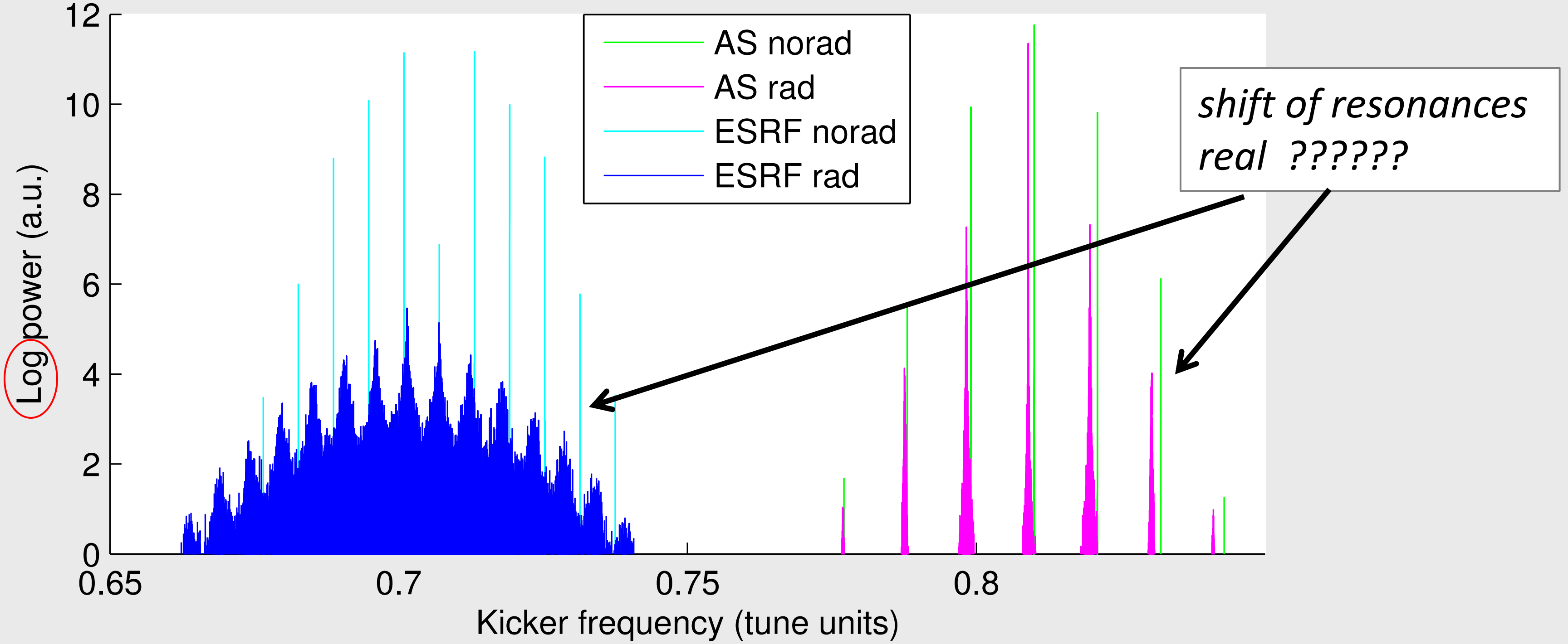
Which effects influence the spin motion and resonance BW?

3. Radiation effects

-- > Add radiation damping and diffusion (from AT tracking) to equation (1)

-- > FFT :

-- > Resonances are broadened



Which effects influence the spin motion and resonance BW?

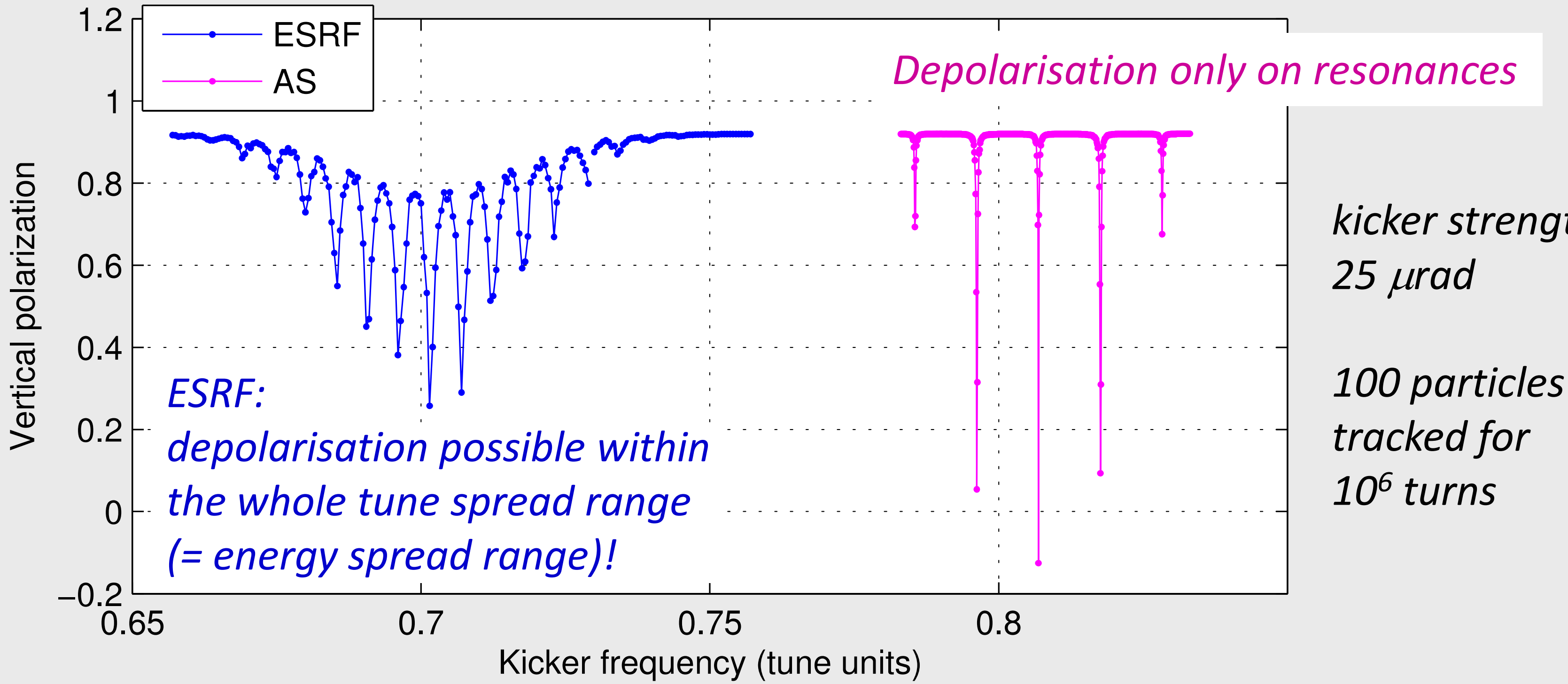
3. Radiation effects

-- > Add radiation damping and diffusion (from AT tracking) to equation (1)

-- > FFT

-- > Spin tracking (FESTA):

-- > Resonances are broadened



kicker strength = 25 μrad

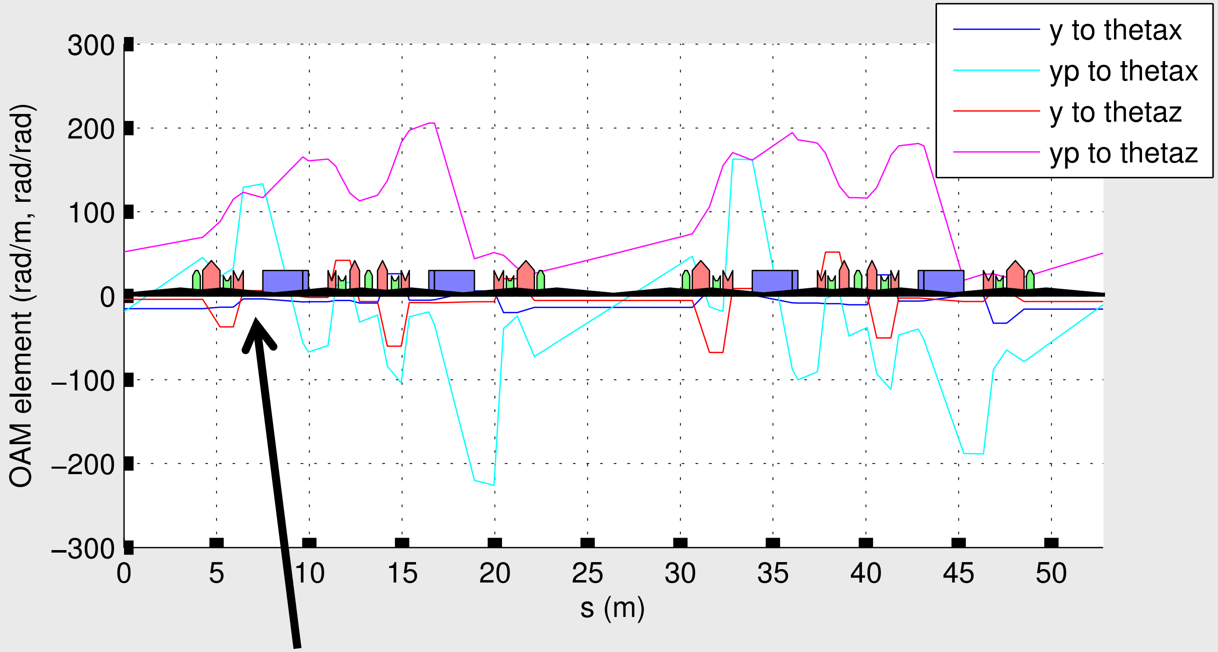
100 particles tracked for 10^6 turns

Which effects influence the spin motion and resonance BW?

4. Effects of quadrupoles (1)

Need to consider the effect of orbital offsets in the quadrupoles -- > additional spin rotation

-- > We construct an "Orbit Angle Matrix" (OAM) that represents a sort of spin response function depending on the position in the lattice:



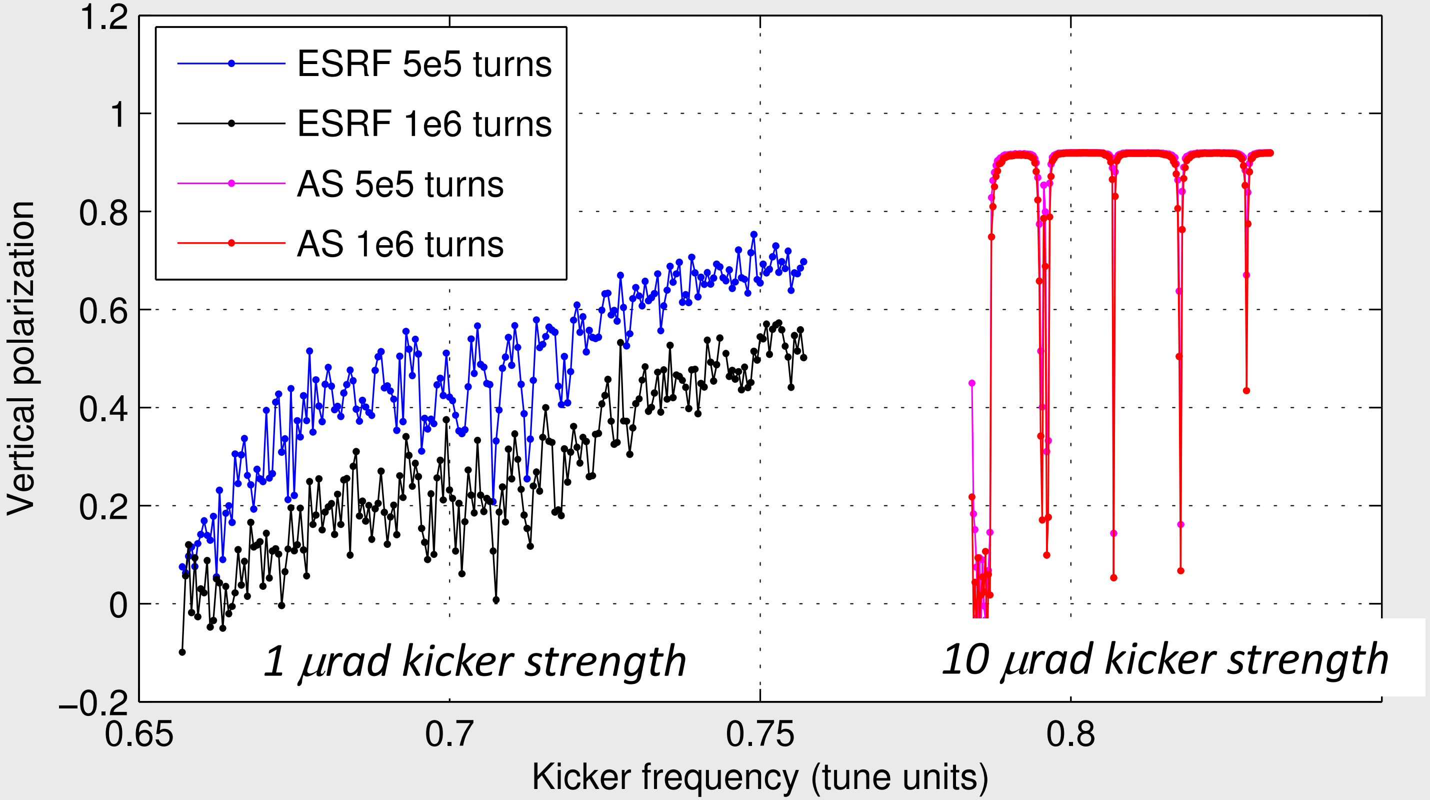
shaker position at ESRF

OAM links the initial phase space coordinates of any particle to the spin angle increment it will receive while travelling for N turns along the orbit.

Which effects influence the spin motion and resonance BW?

4. Effects of quadrupoles (2)

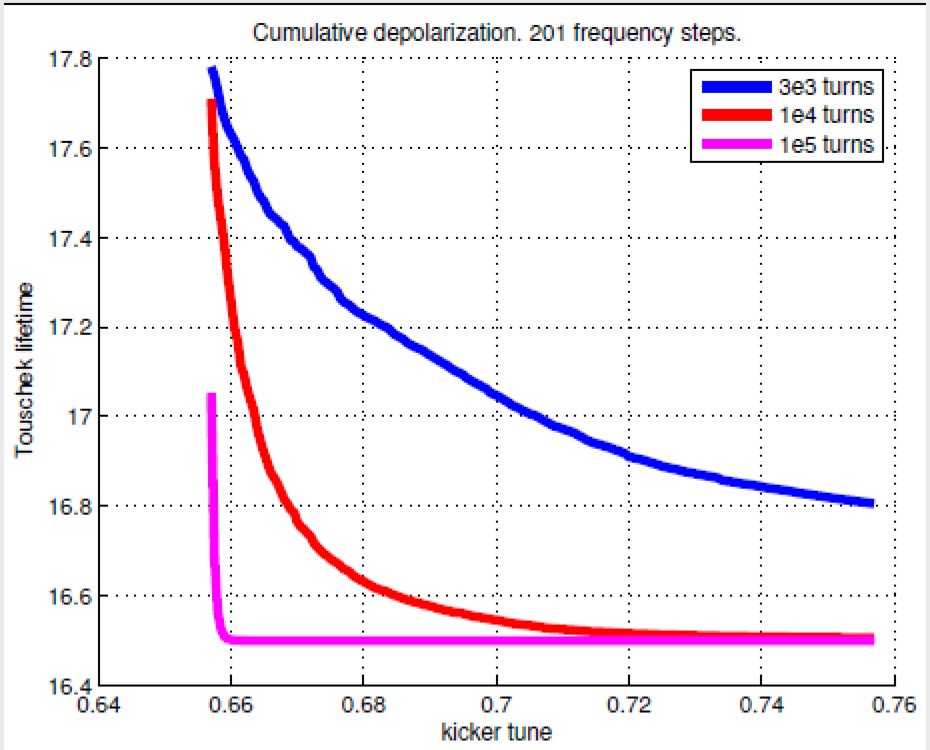
FESTA spin tracking results including OAM for quadrupole effects for ESRF and AS :



- *Resonances are broadened and effect of kicker strength is amplified.*
- *Vertical betatron tune also amplifies the resonances.*

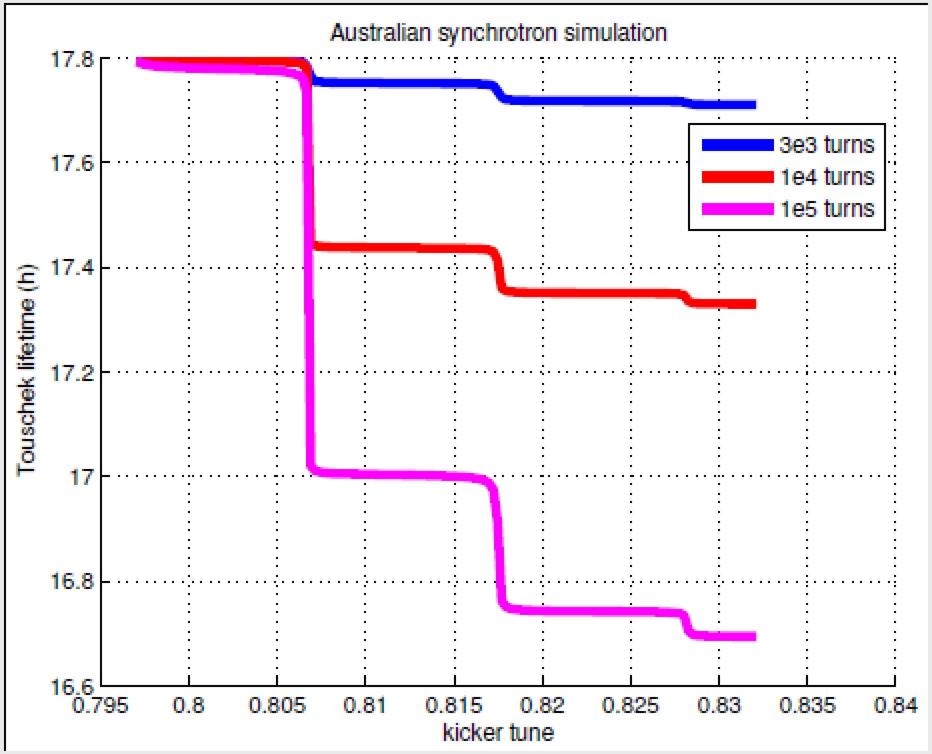
Simulated Touschek-lifetime during excitation with kicker

ESRF



1 μrad kicker strength

Australian Synchrotron



10 μrad kicker strength

Each point starts with the maximum possible polarisation (92%)
(which is not the case in the real experiments! so no exact quantitative comparison of these plots with the data possible)

Conclusions

Three effects seem to influence the quality and detectability of the depolarisation resonance:

1. *Synchrotron oscillations*
   -> B-factor determines the spin tune sideband densities and amplitudes

$$B = \frac{\nu_{sp}\sigma_\delta}{\nu_s}$$

	ESRF	**SPRING8**	**APS**	**Diamond**	**Bessy**	**LEP**	**SPEAR3**	**ALBA**	**SLS**	**AS**	**ANKA**	**ESRF S28**
spin tune	*13.707*	*18.276*	*15.992*	*6.842*	*3.860*	*130.000*	*6.810*	*6.810*	*5.450*	*6.810*	*5.673*	*13.707*
energy spread	*1.06E-03*	*1.00E-03*	*1.01E-03*	*9.62E-04*	*6.60E-04*	*1.20E-03*	*1.20E-03*	*1.01E-03*	*8.60E-04*	*1.00E-03*	*1.00E-03*	*1.01E-03*
synchrotron tune	*5.43E-03*	*7.78E-03*	*7.20E-03*	*3.37E-03*	*1.50E-03*	*1.20E-01*	*8.00E-03*	*8.51E-03*	*6.25E-03*	*1.04E-02*	*9.94E-03*	*3.45E-03*
B	***2.68***	***2.35***	***2.24***	***1.95***	***1.70***	***1.30***	***1.02***	***0.81***	***0.75***	***0.65***	***0.57***	***4.01***

2. *Radiation effects -> lead to line broadening and amplitude reduction*

3. *Orbital offsets in the quadrupoles -> further line broadening.*

see also:

N. Carmignani et al. "Modeling and Measurements of Spin Depolarsiation", IPAC 2015, MOPWA013.